Inverse Trigonometric Functions

Functions	Domain (x)	Range (y)
$y = \sin^{-1}x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$y = \tan^{-1}x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$y = \operatorname{cosec}^{-1}x$	$R - (-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$y = \sec^{-1}x$	$R - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$y = \cot^{-1}x$	R	$(0, \pi)$

✓ Properties of inverse trigonometric functions

$\sin^{-1}(\sin x) = x \quad : \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\cos^{-1}(\cos x) = x \quad : \quad x \in [0, \pi]$

$\tan^{-1}(\tan x) = x \quad : \quad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\cot^{-1}(\cot x) = x \quad : \quad x \in (0, \pi)$

$\sec^{-1}(\sec x) = x \quad : \quad x \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$

$\operatorname{cosec}^{-1}(\operatorname{cosec} x) = x \quad : \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

$\sin^{-1}x\left(\frac{1}{x}\right) = \operatorname{cosec}^{-1}x \quad : \quad x \geq 1$ OR $x \leq -1$

$\cos^{-1}x\left(\frac{1}{x}\right) = \sec^{-1}x \quad : \quad x \geq 1$ OR $x \leq -1$

$\tan^{-1}x\left(\frac{1}{x}\right) = \cot^{-1}x \quad : \quad x > 0$

$\tan^{-1}x\left(\frac{1}{x}\right) = -\pi + \cot^{-1}x \quad : \quad x < 0$

R = Real Numbers

$\sin x(\sin^{-1}x) \quad : \quad x \in [-1, 1]$

$\cos x(\cos^{-1}x) \quad : \quad x \in [-1, 1]$

$\tan x(\tan^{-1}x) \quad : \quad x \in R$

$\cot x(\cot^{-1}x) \quad : \quad x \in R - (-1, 1)$

$\sec x(\sec^{-1}x) \quad : \quad x \in R - (-1, 1)$

$\operatorname{cosec} x(\operatorname{cosec}^{-1}x) \quad : \quad x \in R$

$\sin^{-1}(-x) = -\sin^{-1}x \quad : \quad x \in [-1, 1]$

$\cos^{-1}(-x) = \pi - \cos^{-1}x \quad : \quad x \in [-1, 1]$

$\tan^{-1}(-x) = -\tan^{-1}x \quad : \quad x \in R$

$\cot^{-1}(-x) = \pi - \cot^{-1}x \quad : \quad x \in R$

$\sec^{-1}(-x) = \pi - \sec^{-1}x \quad : \quad |x| \geq 1$

$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x \quad : \quad |x| \geq 1$

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2} \quad : \quad x \in [-1, 1]$

$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2} \quad : \quad x \in R$

$\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2} \quad : \quad |x| \geq 1$

$$\sin^{-1}x + \sin^{-1}y = \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}$$

$$\sin^{-1}x - \sin^{-1}y = \sin^{-1}\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\}$$

$$\cos^{-1}x + \cos^{-1}y = \cos^{-1}\{xy - y\sqrt{1-x^2}\sqrt{1-y^2}\}$$

$$\cos^{-1}x - \cos^{-1}y = \cos^{-1}\{xy + y\sqrt{1-x^2}\sqrt{1-y^2}\}$$

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right) \;:\; xy < 1$$

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right) \;:\; xy > -1$$

$$2\sin^{-1}x = \sin^{-1}(2x\sqrt{1-x^2}) \;:\; -\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$$

$$2\cos^{-1}x = \cos^{-1}(2x^2-1) \;:\; 0 \le x \le 1$$

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right) \;:\; -1 < x < 1$$

$$2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right) \;:\; -1 \le x \le 1$$

$$2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) \;:\; x \ge 0$$

$$2\tan^{-1}x = \pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right) \;:\; \text{if } x > 1$$

$$3\sin^{-1}x = \sin^{-1}(3x-4x^3) \;:\; -\frac{1}{2} \le x \le \frac{1}{2}$$

$$3\cos^{-1}x = \cos^{-1}(4x^3-3x) \;:\; \frac{1}{2} \le x \le 1$$

$$3\tan^{-1}x = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) \;:\; -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$$

Expression	Substitution
a^2+x^2 OR $\sqrt{a^2+x^2}$	$x = a\tan\theta$ OR $x = a\cot\theta$
a^2-x^2 OR $\sqrt{a^2-x^2}$	$x = a\sin\theta$ OR $x = a\cos\theta$
x^2-a^2 OR $\sqrt{x^2-a^2}$	$x = a\sec\theta$ OR $x = a\,\mathrm{cosec}\,\theta$
$\sqrt{\frac{a-x}{a+x}}$ OR $\sqrt{\frac{a+x}{a-x}}$	$x = a\cos 2\theta$
$\sqrt{\frac{a^2-x^2}{a^2+x^2}}$ OR $\sqrt{\frac{a^2+x^2}{a^2-x^2}}$	$x^2 = a^2\cos 2\theta$
$\sqrt{\frac{x}{a-x}}$ OR $\sqrt{\frac{a-x}{x}}$	$x = a\sin^2\theta$ OR $x = a\cos^2\theta$
$\sqrt{\frac{x}{a+x}}$ $\sqrt{\frac{a+x}{x}}$	$x = a\tan^2\theta$ OR $x = a\cot^2\theta$

$$\sin^{-1}x = \cos^{-1}(\sqrt{1-x^2}) = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$
$$= \cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) = \mathrm{cosec}^{-1}\left(\frac{1}{x}\right)$$

$$\cos^{-1}x = \sin^{-1}(\sqrt{1-x^2}) = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$$
$$= \cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) = \sec^{-1}\left(\frac{1}{x}\right) = \mathrm{cosec}^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$$

$$\tan^{-1}x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$$
$$= \cot^{-1}\left(\frac{1}{x}\right) = \sec^{-1}(\sqrt{1+x^2}) = \mathrm{cosec}^{-1}\left(\frac{\sqrt{1+x^2}}{x}\right)$$